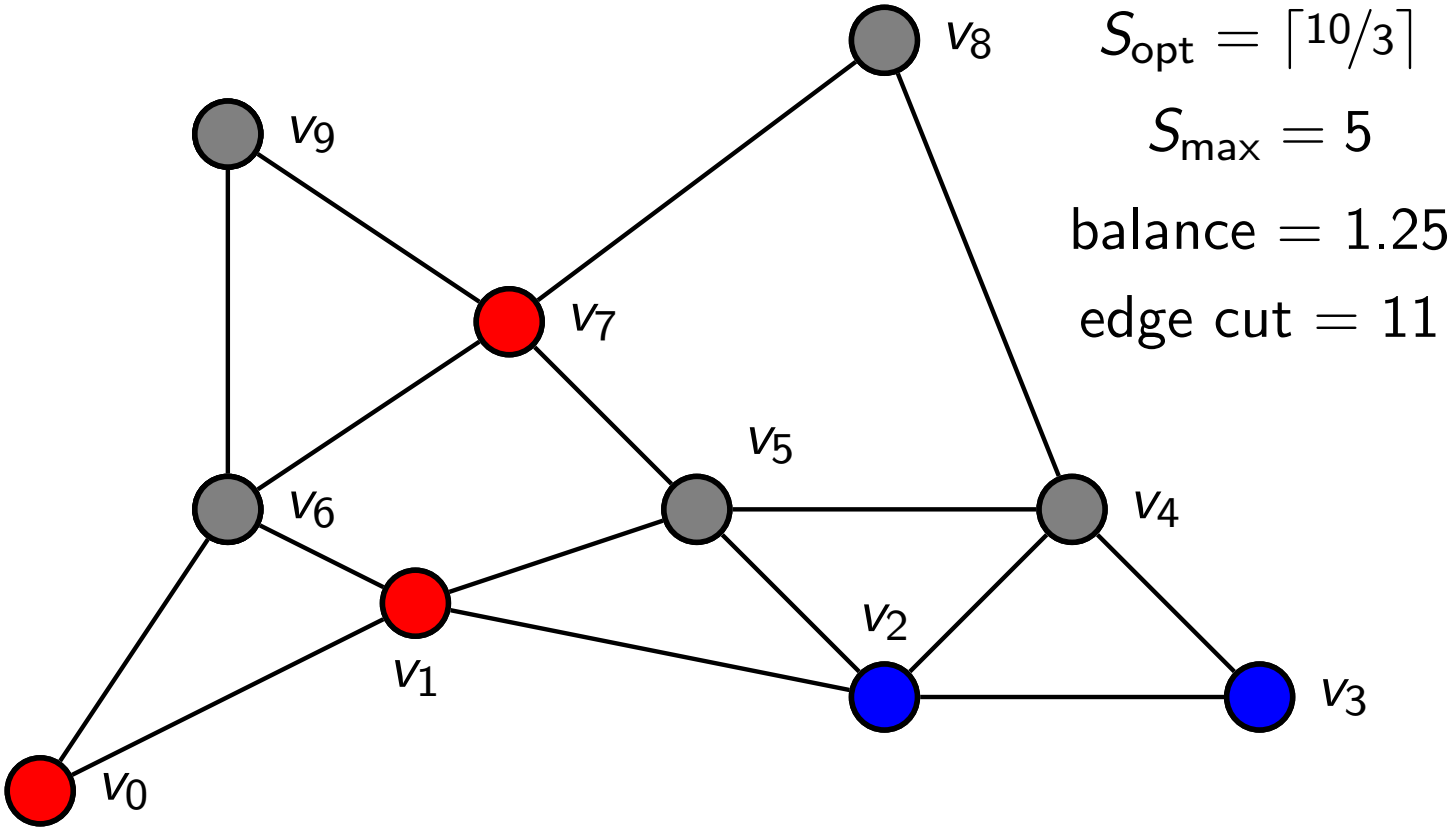
Engineering a Scalable High Quality Graph Partitioner

Manuel Holtgrewe, Peter Sanders, Christian Schulz

Institute for Theoretical Computer Science, Algorithmics II

Novemeber 24, 2009

Graph Partitioning



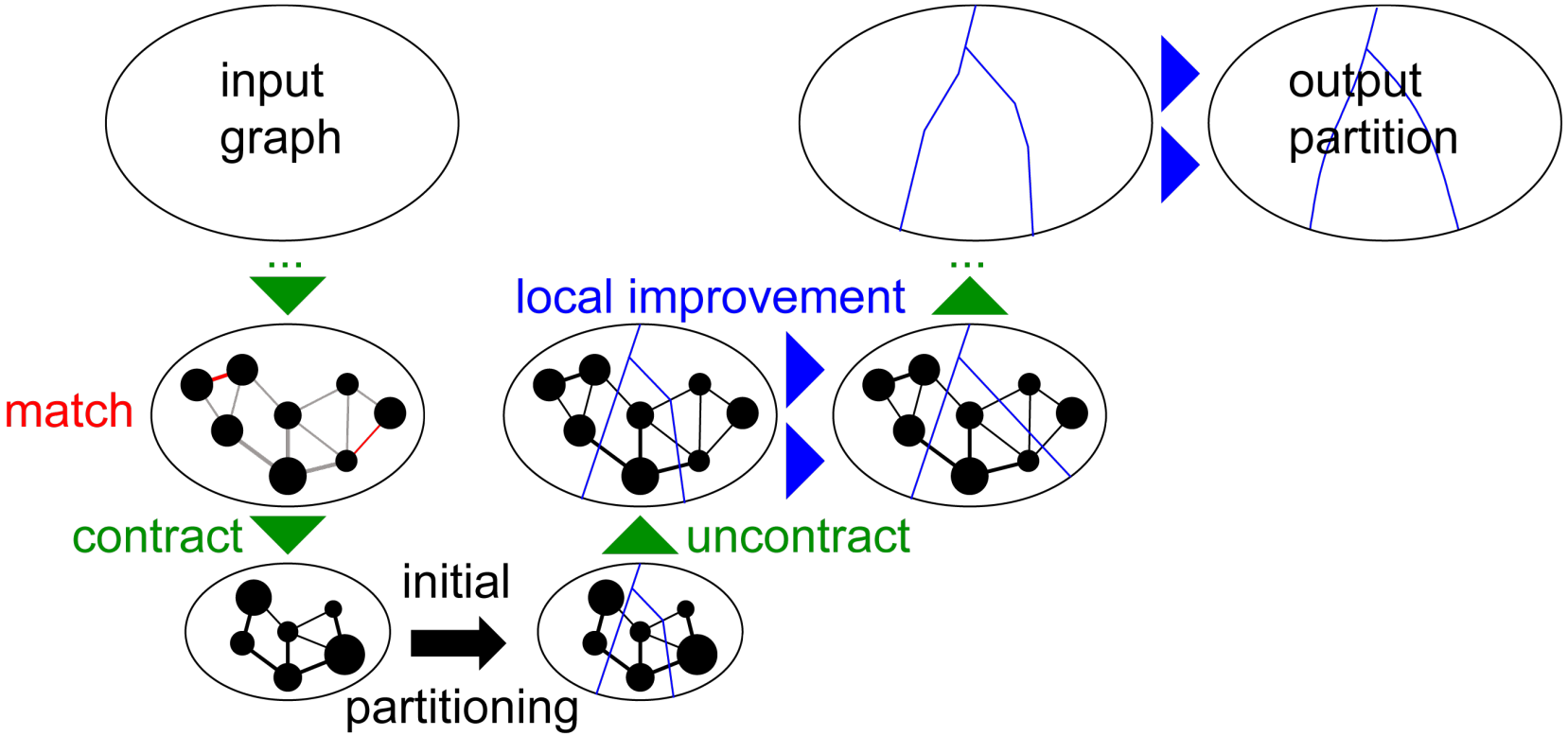
$S_{\text{opt}} = \lceil 10/3 \rceil$

$S_{\text{max}} = 5$

balance $= 1.25$

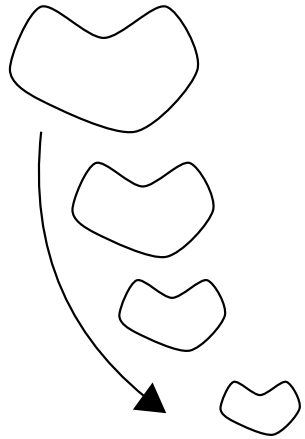
edge cut $= 11$

Well-Known Multi-Level Approach



Match and Contract

- Sequential Matching: GPA [Maue, Sanders, 2007].
  1. 1st: greedy algorithm finds heavy paths and cycles
  2. 2nd: optimal matching on those using dynamic programming
- GPA, Greedy, HEM achieve half-approx. in worst case
- GPA gives considerably better results (empirically)
- Parallel Matching running on the gap graph: Implementation of Manne and Bisseling's sketch [Manne, Bisseling, 2007].
- Use edge ratings for priorization



Edge Ratings - Few Basic Principles

- good to contract heavy edges because this decreases the cut size
- avoid clusters with many outgoing edges
- contract light nodes to keep the node weight at any level reasonably uniform.

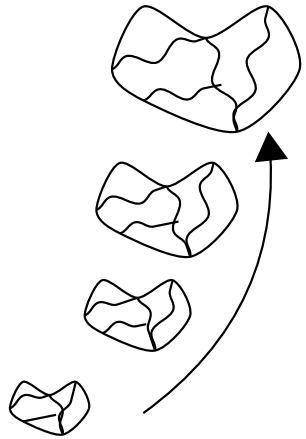
weight	"number of edges in original graph"
expansion	edge weight divided by the number of vertices
expansion*	edge weight divided by the product of number of vertices
inner-outer	edge weight divided by number of edges out of cluster

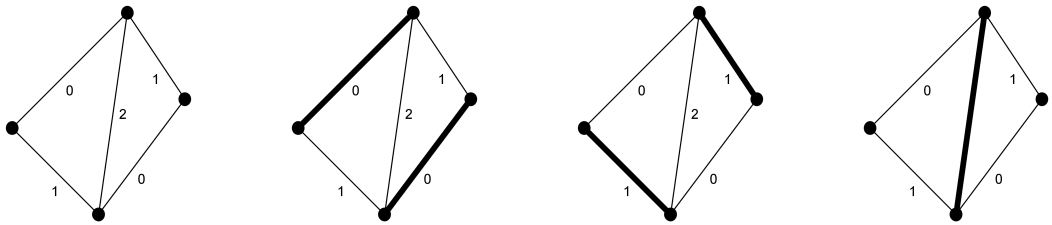
Initial Partitioning

- Every PE can perform initial partitioning using different seeds
- Compared PARTY, PMETIS, and SCOTCH
- SCOTCH yielded best results
- we will try other partitioners as well

Uncontract and Local Improvement

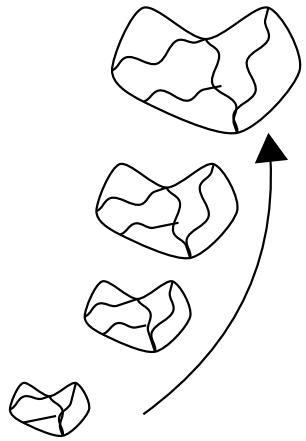
- we adopt basic approach from [Fiduccia, Mettheyses, 1982] (linear time)
- basic idea: at any time, each PE may work on one pair of neighboring blocks
- local search constrained to moving nodes between these two blocks graph
- assign pairs of blocks to PEs, through edge colorings of *quotient graph Q*



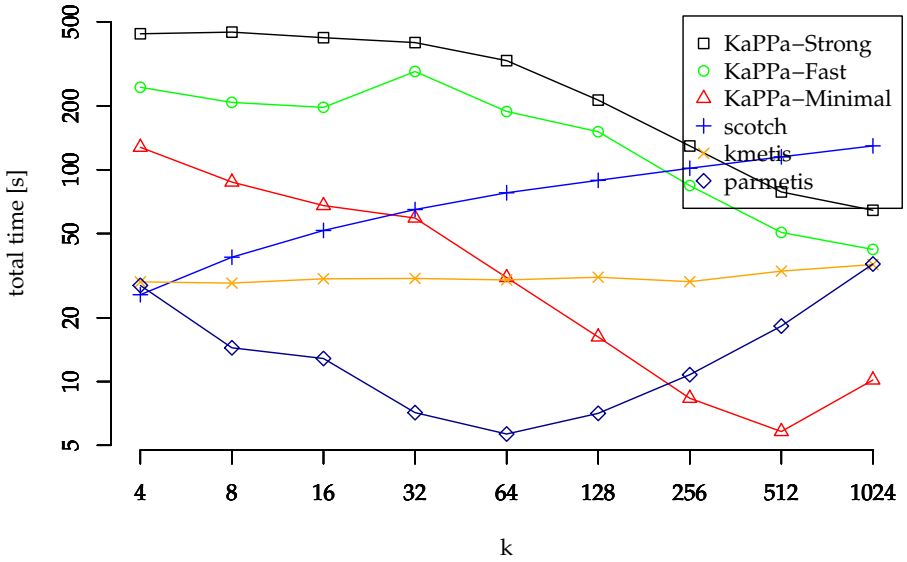


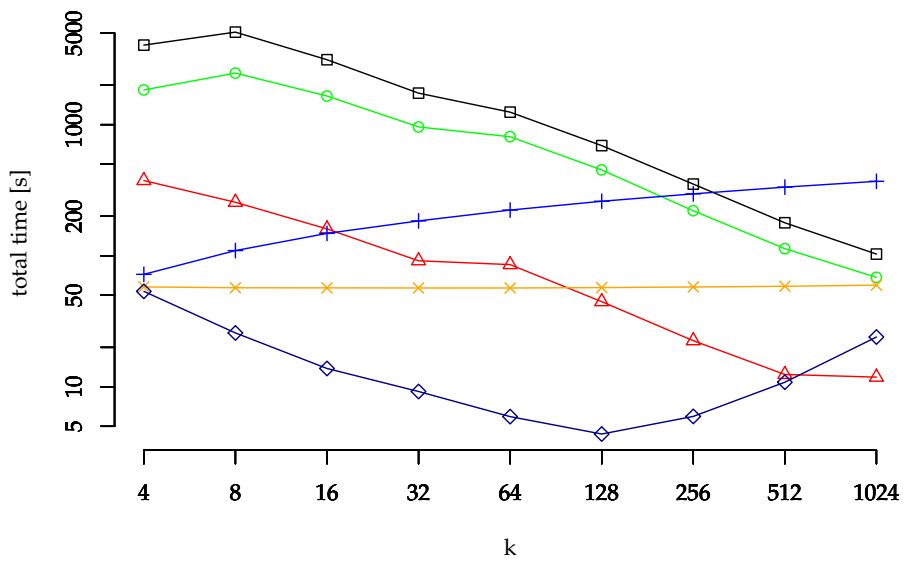
Uncontract and Local Improvement

- BFS to reduce communication volume
- Queue Selection Strategies of local search improved quality
  1. use queue promising larger gain (TopGain)
  2. use queue of heavier block (MaxLoad) if one block is overloaded



Scalablity





- Streetnetwork Europe ($|V| = 18M, |E| = 44M$)
- DelaunayGraph ($|V| = 33M, |E| = 100M$)

Quality - Walshaw Benchmark

Five percent case highlights

Graph	2		4		8		16		32		64	
wing	+ 826	**770**	** 1734	**1636**	* 2632	**2551**	* 4106	**4015**	* 6063	**5806**	+ 8300	**7991**
brack2	** **660**	660	** **2739**	2755	* **6776**	6883	+ **11557**	11558	* 17617	**17529**	+ 26555	**26281**
finan512	** **162**	162	** **324**	324	** **648**	648	* **1296**	1296	** **2592**	2592	** 10909	**10560**
fetooth	** 3785	**3773**	* **6863**	7027	+ **11498**	11957	* **17509**	18090	* 25641	**25624**	+ 35795	**35476**
ferotor	** **1955**	1957	+ **7031**	7520	* **12643**	12678	** **20098**	20263	+ 31611	**31576**	+ 47186	**46608**
598a	** 2344	**2336**	+ **7837**	7978	** **15794**	16031	* **25782**	26257	** **39478**	40179	** **58180**	58307
feocean	** **311**	311	** **1688**	1704	* **3952**	4019	+ **7671**	7838	+ 12953	**12746**	+ **20660**	21784
144	* 6502	**6362**	+ 15313	**15250**	** **25529**	25611	+ **38182**	38478	+ **57202**	57354	+ 80653	**80257**
wave	** 8613	**8563**	+ **16780**	17306	+ **28753**	29776	+ **42810**	43791	** **62382**	63675	** **86867**	87654
m14b	** 3844	**3802**	* **13124**	13285	** **25701**	26153	+ **42644**	43747	* **66845**	67551	+ **99460**	100183
auto	* 9587	**9450**	+ **25805**	26097	** **44915**	48174	+ **76500**	80495	** **121988**	124251	** **174173**	174904

Future Work

- Base case $\Rightarrow$ Meyerhenke's code?
- Back to sequential partitioning
- Other pairwise improvers? Flow? Diffusion?
- Improve overall search strategies
- Put every aspect of multilevel graph partitioning on trial
- More information: http://arxiv.org/pdf/0910.2004

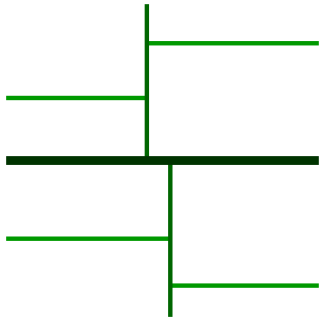
Thank you for your interest. Any questions?

Building Parts

- Recursive Coordinate Bisection as a prepartitioner
- Sequential and parallel algorithms for approximating maximal weight matchings
- Target functions for edges (*edge ratings*)
- Parallel contraction code
- Initial partitioning library (SCOTCH [Pellegrini, Roman, 1996])
- Pairwise refinement algorithm

Recursive Coordinate Bisection

- Similar to Bentley's *k*d-trees
- Parallel Quick-Select
- Could be improved using Parallel Randomized Selection algorithm by Blume et al.



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